Alexandria Journal of Mathematics (ISSN 2090-4320)
Volume2 -Number1 - June 2011

The Cranked Nilsson Model and the Structure of the Even-Even Deformed Nuclei in the sd-Shell

Kh. A. Kharroub[1], M. O. Shaker[2], A. M. Farag[2]

S. B. Doma[3]

[1] Department of Mathematics, Faculty of Science, Lebanese University, Beirut, Lebanon.
[2] Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt.
[3] Department of Mathematics, Faculty of Science, Alexandria University, Moharram Bay, Alexandria, Egypt. E-mail address :sbdoma@sci.alex.edu.eg

Abstract
The electric quadrupole moment, the total ground-state energy, the nuclear moment of inertia, the liquid drop inertia, the liquid drop energy, and the Strutinsky inertia of the even-even nuclei in the sd-shell; namely: ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S and ^{36}Ar are calculated by applying the cranked Nilsson model and the single-particle Schrödinger fluid model. The obtained results of the electric quadrupole moment for these nuclei are in good agreement with the corresponding experimental values. Moreover, the obtained results of the cranking model-moment of inertia, by using the single-particle Schrödinger fluid, for these nuclei are also in good agreement with the corresponding experimental values.

Keywords: Even-even deformed nuclei, Cranked Nilsson-Strutinsky model, single particle Schrödinger fluid, moment of inertia, electric quadrupole moment.

1. Introduction
A large number of models have been developed to get insight into the spectroscopic properties of nuclei. One of the most successful models that deals with axially symmetric deformed nuclei is the deformed shell model (Nilsson model) [1].

In $N = Z$ nuclei, neutrons and protons occupy the same orbitals, and thus can have the largest probability to interact with each other which leads to neutron-proton pairing. It is well known that the study of $N = Z$ nuclei is the domain which is expected to give the most relevant information about the properties of the neutron-proton (n-p) interaction. An interesting question for the yrast band properties is whether the expected strong n-p interaction will modify the rotational-alignment picture in $N = Z$ nuclei. It has been suggested using cranking approaches in a single-j shell (see the references cited in ref. [2]) that the rotational-alignment properties can be modified by the residual n-p interaction. The main difficulty in extending the study of $N = Z$ nuclei to high spins is that their population has extremely low cross-sections in the small number of available reactions.

On the other hand, the cross-shell excitation costs a large amount of energy, leading to high excitation energy of several MeV for the 2^+-state. In even-even nuclei with one closed and one open shell it does cost significantly less energy to produce this excited state, namely the energy to break a pair of nucleons in the open shell, about 1 to 1.2 MeV [3].

Despite the difficulties, progress in the development of large γ-ray arrays and associated ancillary detectors, and refinements of the data processing techniques, has allowed recent advance in the knowledge of some heavier $N = Z$ nuclei (see, for example, refs. [2, 4, 5, 6]).

Until the present a major activity in the study of shape-phase transitions for nuclei in the ground-state at zero temperature is carried out within the interacting boson model (IBM) (see, for example, [7, 8] and references therein). The model naturally incorporates different symmetry limits associated with specific nuclear properties [9]. While the IBM can be easily extended to a thermodynamical limit $N \rightarrow \infty$, which is well suitable for the study of phase transitions, the analysis is rather oversimplified. For example, the model does not take into account the interplay between single-particle and collective degrees of freedom in even-even nuclei.

In the Nilsson-Strutinsky approach [10], the total energy is split into an average part, parameterized by a macroscopic expression, and a fluctuating part extracted from the variation of the level density around the Fermi surface. The microscopic part includes the Strutinsky shell correction and the pairing energy. In the Cranked Nilsson Strutinsky (CNS) model [10], we calculate the intrinsic quadrupole moments from proton single-particle wave functions at appropriate equilibrium deformations, such that neutrons have no contribution to this moment.

The concept of the single-particle Schrödinger fluid [11-13] is based on generating and adopting a fluid-mechanical interpretation of the time-dependent Schrödinger equation by suitably choice of the single-particle wave function. It gives an accurate method for calculating the nuclear moment of inertia of an axially deformed nucleus in framework of the cranking model, but for the rigid-body model further modifications are needed to achiev good agreement with the corresponding experimental values [13-15].

In the present paper we are dealing with the properties of the even-even deformed nuclei in the sd-shell; namely the nuclei: ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S and ^{36}Ar, from the point of view of the deformation structure. Accordingly, we have applied the single-particle Schrödinger fluid model to calculate the reciprocal moments of inertia of the mentioned five nuclei for both of the rigid-body model and the cranking model. Furthermore, we have applied the cranked Nilsson Strutinsky model to calculate the electric quadrupole moment, the total ground-state energy, the liquid drop inertia, the liquid drop energy, and the Strutinsky inertia of the five mentioned nuclei.

2. Cranked Nilsson – Strutinsky Model

The single particle Hamiltonian, H for the cranked Nilsson model assumes the form [16]

$$H = H^{(0)} + H^{(1)} - \omega j_x, \tag{2.1}$$

$$H^{(0)} = \frac{p^2}{2m} + \frac{1}{2} m\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right), \tag{2.2}$$

where $H^{(0)}$is the triaxial Nilsson Hamiltonian [17].

The oscillator frequencies ω_x, ω_y and ω_z assume the compact form [18]

$$\omega_j = \omega_0(\varepsilon,\gamma)\left[1 - \frac{2}{3}\varepsilon \cos\left(\gamma + \frac{2\pi\, \nu_j}{3}\right)\right], j \in \{x, y, z\} \tag{2.3}$$

with $\nu_x = 1, \nu_y = -1$ and $\nu_z = 0$. The calculations are carried out in the stretched coordinate system [19, 20], $\xi = x\sqrt{m\omega_x/\hbar}$, $\eta = y\sqrt{m\omega_y/\hbar}$ and $\zeta = z\sqrt{m\omega_Z/\hbar}$. The parameters ε and γ are the quadrupole deformation degrees of freedom. The quantity ε is related to the deformation parameter β by the relation $\varepsilon = \frac{3}{2}\sqrt{\frac{5}{4\pi}}\beta$.

The second term in the right hand side of equation (2.1) is given by

$$H^{(1)} = 2\hbar\omega_0\sqrt{\frac{4\pi}{g}}\rho^2 \varepsilon_4 V_4 + V', \tag{2.4}$$

where

$$\rho^2 = \frac{m}{\hbar}\left(\omega_x x^2 + \omega_y y^2 + \omega_z z^2\right). \tag{2.5}$$

Here ρ^2 is the stretched square radius.

The potential V_4 in equation (2.4) is the hexadecapole potential and is given by [21, 22]

$$V_4 = \frac{1}{6}(5\cos^2\gamma + 1)Y_{4,0}\frac{1}{12}\sqrt{30}\sin 2\gamma\left(Y_{4,2} + Y_{4,-2}\right) + \frac{1}{12}\sqrt{70}\sin^2\gamma\left(Y_{4,4} + \right.$$
$$\left. Y4,-4. \right. \tag{2.6}$$

In equation (2.4) the potential V' is given by

$$V' = -\kappa\,\hbar\omega_0^0\{\boldsymbol{\ell}_t.\mathbf{s} + \mu(\boldsymbol{\ell}_t^2 - \langle\boldsymbol{\ell}_t^2\rangle_N)\}. \tag{2.7}$$

In the above equation t refers to the stretched coordinates, $\xi = x\sqrt{m\omega_x/\hbar}$ etc., the parameters κ and μ might either be given the same values for each shell or, alternatively, as indicated in (2.7), they can be made dependent on the main oscillator quantum number $N = N_t$. The parameter ω_0^0 is the oscillator strength (or is the value of ω_0 when β = 0) and ε_4 in equation (2.4) refers to the hexadecapole deformations degree of freedom.

3. The Electric Quadrupole Moment of Deformed Nuclei

In [23] the nuclear electric quadrupole moment is a parameter which describes the effective ellipsoidal shape of the nuclear charge distribution. The deviation from spherical symmetry is given by the electric quadrupole moment and is denoted by the symbol Q. The definition of Q is given in terms of the operator Q_0, which is expressed in the form:

$$Q_0 = \int \rho_{ch}(r)(3z^2 - r^2)d\tau, \tag{3.1}$$

with the z-axis along the axis of symmetry defined by nuclear spin. The expression in (3.1) is the average of $(3z^2 - r^2)$ taken over the charge density distribution:

$$Q_0 = Z(3\langle z^2\rangle - \langle r^2\rangle), \langle r^2\rangle = \langle x^2\rangle + \langle y^2\rangle + \langle z^2\rangle. \tag{3.2}$$

Here Z is the total nuclear charge, in units of e. The units of Q_0 are cm^2 or barns ($1\ \text{barn} = 10^{-24}cm^2$). For a spherical charge distribution

$$\langle x^2\rangle = \langle y^2\rangle = \langle z^2\rangle. \tag{3.3}$$

If we substitute the above expression in equation (3.2) we get $Q_0 = 0$. If $\langle z^2\rangle > \frac{1}{3}\langle r^2\rangle$ the shape becomes prolate ellipsoid for which $Q_0 > 0$ and if $\langle z^2\rangle < \frac{1}{3}\langle r^2\rangle$ we get oblate ellipsoid for which $Q_0 < 0$.

In quantum mechanics: ρ is replaced by $\Psi^*\Psi$, where Ψ is the wave function which leads to the result for the nucleus of spin J that measures nuclear electric quadrupole moments. For $J > 0$, the symbol Q is defined by

$$Q = \frac{2J-1}{2(J+1)}Q_0. \tag{3.4}$$

For $J = 0, \frac{1}{2}: Q_0 = 0$ which implies that there is no rotation since there is no symmetry axis defined. In summary: in order to learn something about the shape of the nucleus, we have to determine its intrinsic quadrupole moment, Q_0, defined with respect to the body fixed frame. For a nucleus which has both N and Z value magic numbers, the nuclear charge is symmetrically distributed and the shape is spherical, $Q_0 = 0$. If the charge density of the nucleus is concentrated along the z-axis (symmetry axis of the particle), the term proportional to $3z^2$ dominates, Q_0 is positive and the shape is prolate (cigar shape).

There are two important relations combines between the intrinsic quadrupole moment Q_0, Q_s and deformation parameter β follows:

$$Q_s = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} Q_0, \tag{3.5}$$

$$Q_0 = \frac{3}{\sqrt{5\pi}} ZR^2 \beta(1 + 0.36\beta), \tag{3.6}$$

with $R = R_0 A^{1/3}$.

The methods of calculating the total ground-state energy, the liquid drop inertia, the liquid drop energy, the volume conservation factor and the Strutinsky inertia are well-known and can be found in [10].

4. Single-Particle Schrödinger Fluid

The most important property distinguishing non spherical from spherical nuclei is the presence of rotational energy levels in the non spherical nuclei. The study of the velocity fields for the rotational motion of the nucleon in a deformed nucleus led to the formulation of the concept of the single-particle Schrödinger fluid [12]. Since the Schrödinger fluid theory is at present an independent particle model, the cranking model approximation for the velocity fields and the moments of inertia play the dominant role in this theory.

In the static part of the single particle Schrödinger fluid the nucleon is assumed to move in an axially deformed potential of the nucleus, which is chosen to be the anisotropic oscillator with $\omega_x = \omega_y \neq \omega_z$.

The polar form of the time-dependent K^{th} -single particle wave function is given by [11]

$$\Psi_k(\mathbf{r}, \alpha(t), t) = \Phi(\mathbf{r}, \alpha(t)) \exp\left\{-i\frac{M}{\hbar} S_K(\mathbf{r}, \alpha(t)) - \frac{i}{\hbar}\int_0^t \varepsilon_K\left(\alpha(t')\right) dt'\right\}. \tag{4.1}$$

Here $\alpha(t)$ represents some time-dependent collective parameters, S is a real function and Φ is a positive real function. In the case of rotation, the parameter $\alpha(t)$ becomes the angle of rotation, θ.

The single-particle Hamiltonian H is α-dependent through its potential and the time-dependent Schrödinger equation

$$\mathrm{H}\big(\mathbf{r},\mathbf{p},\alpha(t)\big)\Psi_k(\mathbf{r},\alpha(t),t) = i\hbar\frac{\partial}{\partial t}\Psi_k(\mathbf{r},\alpha(t),t), \tag{4.2}$$

can be separated into real and imaginary parts, by using Eq. (4.1), and as a result two equations are obtained. The first is the continuity equation

$$\rho\,\nabla.\mathbf{v} + \mathbf{v}.\nabla\rho = -\frac{\partial\rho}{\partial t}, \tag{4.3}$$

where the density $\rho = \Phi^2$ and the irrotational velocity field $\mathbf{v}$ is defined by

$$\mathbf{v} = -\nabla S, \tag{4.4}$$

$$S = \frac{i\hbar}{2M}\ln(\Psi/\,\Psi^*). \tag{4.5}$$

The second equation is

$$\big(H + V_{dyn}\big)\emptyset_i = \varepsilon_i\emptyset_i. \tag{4.6}$$

This is a modified Schrödinger equation through the modified dynamical potential

$$V_{dyn} = -M\left(\frac{\partial S}{\partial t} - \frac{1}{2}v^2\right). \tag{4.7}$$

In addition to the irrotational velocity field $\mathbf{v}$, which has been result from the fluid dynamical equation, other velocity fields which satisfy the continuity equation of the Schrödinger equation occur. Among these velocity fields are the incompressible velocity field, the regular velocity field, the geometric velocity field and the rigid body velocity field.

For rotations, the rigid body velocity field $\mathbf{v}_{rig}$ is defined by

$$\mathbf{v}_{rig} = \mathbf{\Omega}\times\mathbf{r}. \tag{4.8}$$

It is seen that this velocity field is incompressible, regular and also of a geometric type.

In the adiabatic approximation where $\frac{\partial\alpha}{\partial t}\to 0$, the collective kinetic energy of a nucleon in the nucleus is given by [11]

$$T_K = \frac{1}{2}M\int\rho_K\mathbf{v}_T\,.\,(\mathbf{\Omega}\times\mathbf{r})d\tau, \tag{4.9}$$

and the collective kinetic energy T of the nucleus is given by

$$T = \frac{1}{2} M \int \rho_T \mathbf{v}_T . (\mathbf{\Omega} \times \mathbf{r}) d\tau, \tag{4.10}$$

where ρ_T is the total density distribution of the nucleus and $\mathbf{v}_T$ is the total velocity field

$$\mathbf{v}_T = \frac{\sum_{k=occ} \rho_K \mathbf{v}_K}{\sum_{k=occ} \rho_K}. \tag{4.11}$$

5. Moments of Inertia

The following expressions for the cranking-model and the rigid body-model moments of inertia can be easily obtained on the basis of the concept of the single-particle Schrödinger fluid [11, 12]

$$\mathfrak{I}_{cr} = \frac{E}{\omega_0^2} \left(\frac{1}{6+2\sigma}\right) \left(\frac{1+\sigma}{1-\sigma}\right)^{\frac{1}{3}} \left[\sigma^2 (1+q) + \frac{1}{\sigma}(1-q)\right]. \tag{5.1}$$

$$\mathfrak{I}_{rig} = \frac{E}{\omega_0^2} \left(\frac{1}{6+2\sigma}\right) \left(\frac{1+\sigma}{1-\sigma}\right)^{\frac{1}{3}} [(1+q) + \sigma(1-q)]. \tag{5.2}$$

where q is the anisotropy of the configuration, which is defined by

$$q = \frac{\sum_{occ}(n_y+1)}{\sum_{occ}(n_z+1)} \tag{5.3}$$

and E is the total energy

$$E = \sum_{occ}\left[\hbar\omega_x \left(n_x + n_y + 1\right) + \hbar\omega_z (n_z + 1)\right]. \tag{5.4}$$

In equations (5.3) and (5.4) n_x, n_y and n_z are the state quantum numbers of the oscillator. The summations in (5.3) and (5.4) are carried out over all the occupied single-particle states. The method of filling these states is illustrated in [15]. Also, in (5.1) and (5.2) σ is a measure of the deformation of the potential and is defined by

$$\sigma = \frac{\omega_y - \omega_z}{\omega_y + \omega_z}. \tag{5. 5}$$

The frequencies ω_x, ω_y and ω_z are given by [17]

$$\omega_x^2 = \omega_y^2 = \omega_0^2(\delta) \left(1 + \frac{2}{3}\delta\right), \tag{5.6}$$

$$\omega_z^2 = \omega_0^2(\delta) \left(1 - \frac{4}{3}\delta\right), \tag{5.7}$$

$$\omega_0(\delta) = \omega_0^0(\delta)\left(1 - \frac{4}{3}\delta^2 - \frac{16}{27}\delta^3\right)^{-\frac{1}{6}}. \tag{5.8}$$

For the non deformed frequency ω_0^0 we use the one which is given in terms of the mass number A, the number of neutrons N and the number of protons Z by [24]

$$\hbar\omega_0^0 = 38.6\ A^{-\frac{1}{3}} - 127.0\ A^{-\frac{4}{3}} + 14.75\ A^{-\frac{4}{3}}(N - Z) \tag{5.9}$$

The well-known deformation parameter β is related to the parameter δ by the following relation [19]

$$\beta = \frac{2}{3}\sqrt{\frac{4\pi}{5}}\,\delta. \tag{5.10}$$

We note that the cranking-model and the rigid body-model moments of inertia are equal only when the harmonic oscillator is at the equilibrium deformation.

Once the nuclear moment of inertia $\mathfrak{I}$ is known, the energy of the rotational states of a deformed even nucleus is given by [25]

$$E_I = \frac{\hbar^2}{2\mathfrak{I}} I(I+1) + BI^2(I+1)^2, \tag{5.18}$$

where B is a constant determined by the rotation-vibration interaction and I is the nuclear total angular momentum number.

6. Results and Conclusions

In the present paper the calculations of the electric quadrupole moment, the ground-state total energy, the Strutinsky inertia, the liquid drop inertia, the liquid drop energy and the volume conservation factor ω_0/ω_0^0, are carried out for the doubly even deformed nuclei in the sd-shell; namely rhe nuclei: ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S and ^{36}Ar by applying the Cranked Nilsson Model. Moreover, the reciprocal values of the cranked-model and the rigid-body model moments of inertia are calculated for these five nuclei by using the concept of the single particle Schrödinger fluid.

We have also investigated the dependence of the calculated cranked Nilsson characteristics of the five nuclei ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S and ^{36}Ar on the non axiality parameter γ (which is assumed to vary between -15° and 60° in steps of 15°.

Firstly, for the nucleus ^{20}Ne

At fixed quadrupole deformation parameters $\varepsilon = 0.3$ and $\varepsilon_4 = 0$ we have

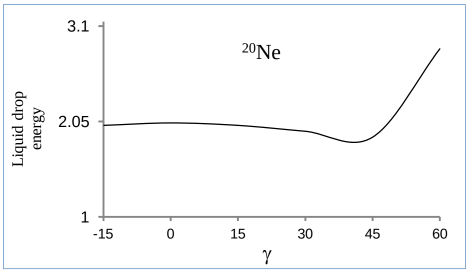

Fig. 1 The L. D. energy as function of the non-axiality parameter γ

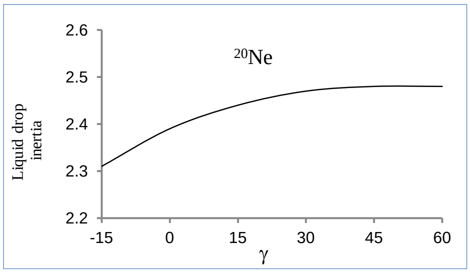

Fig. 2 The L. D. inertia as function of the non-axiality parameter γ

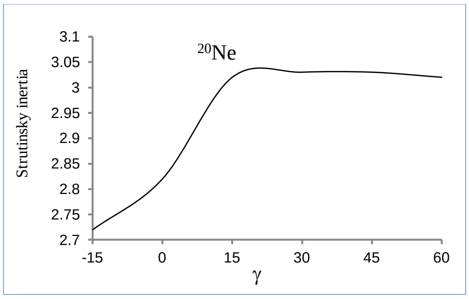

Fig. 3 The Strutinsky inertia as function of γ

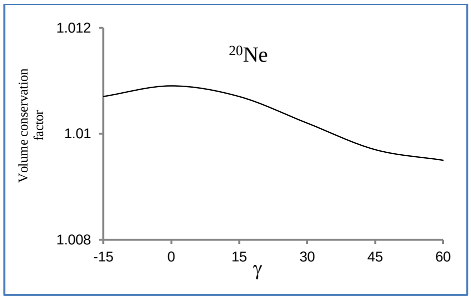

Fig. 4 The volume conservation factor as function of γ

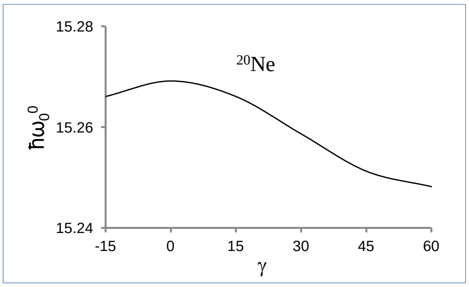

Fig. 5 The oscillator parameter $\hbar\omega_0^0$ as function of γ

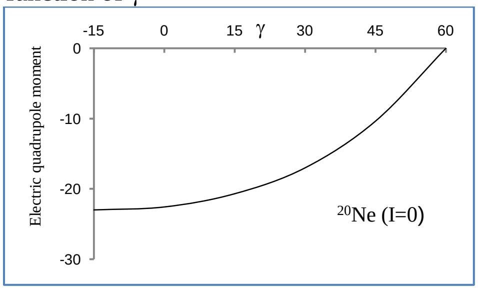

Fig. 6 The electric quadrupole moment as function of γ

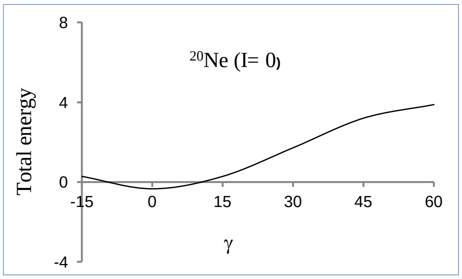

Fig. 7 The total energy as function of γ

Secondly, for the nucleus ^{24}Mg

At fixed quadrupole deformation parameters $\varepsilon = 0.2$ and $\varepsilon_4 = 0$ we have

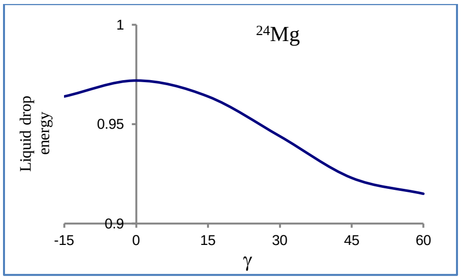

Fig. 8 The Liquid drop energy as function of γ

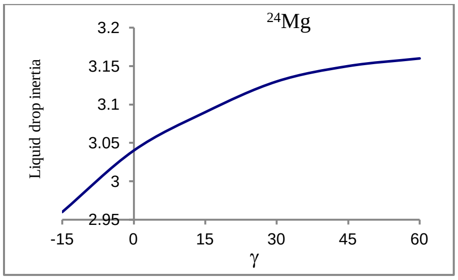

Fig. 9 The Liquid drop inertia as function of γ

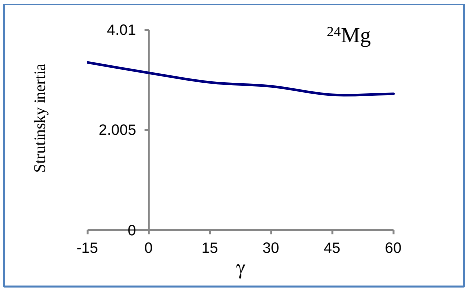

Fig. 10 The Strutinsky inertia as function of γ

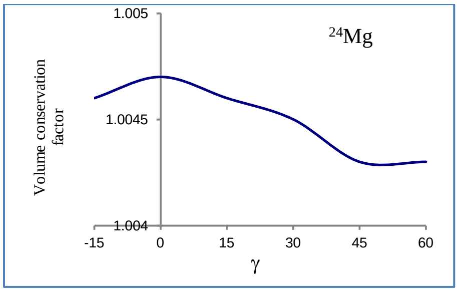

Fig. 11 The volume conservation factor as function of γ

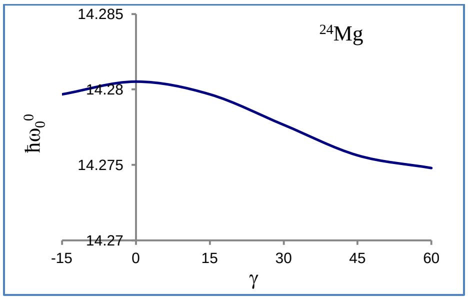

Fig. 12 The oscillator parameter $\hbar\omega_0^0$ as function of γ

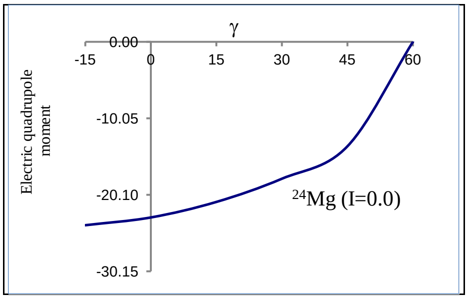

Fig. 13 The electric quadrupole moment as function of γ

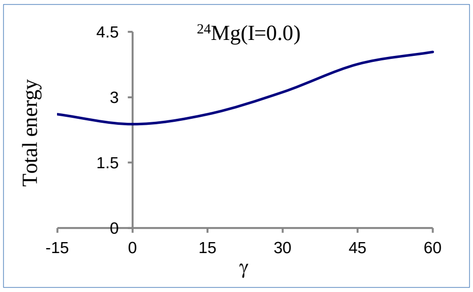

Fig. 14 The total energy as function of γ

Thirdly, for the nucleus ^{28}Si

At fixed quadrupole deformation parameters $\varepsilon = -0.2$ and $\varepsilon_4 = 0.067$ we have

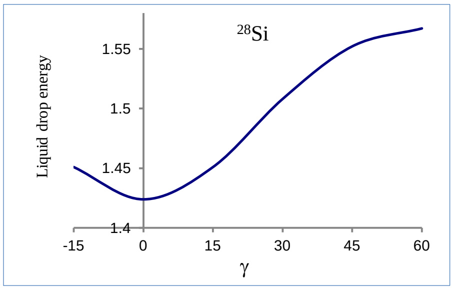

Fig. 15 The L.D.energy as function of γ

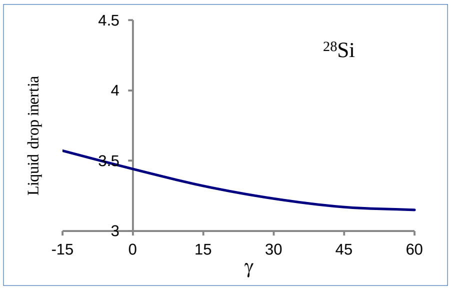

Fig. 16 The L.D.inertia as function of γ

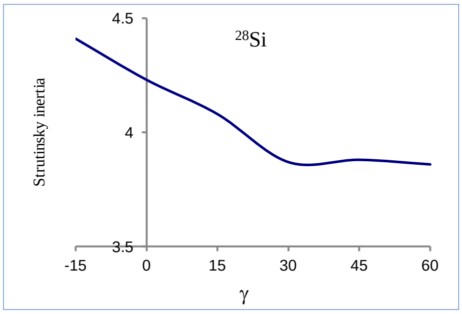

Fig. 17 The Strutinsky inertia as function of γ

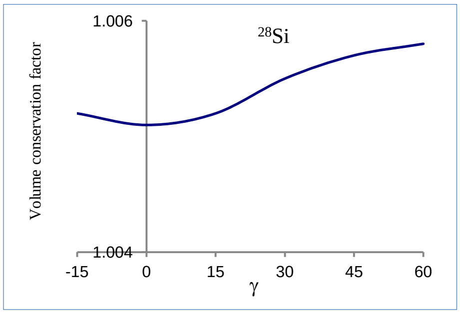

Fig. 18 The volume conservation factor as function of γ

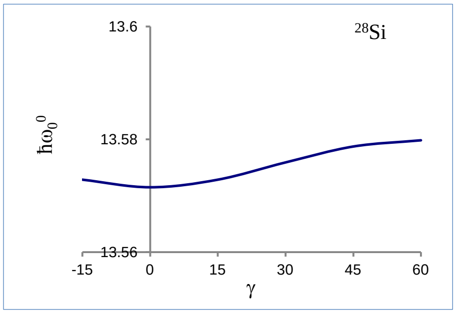

Fig. 19 The oscillator parameter $\hbar\omega_0^0$ as function of γ

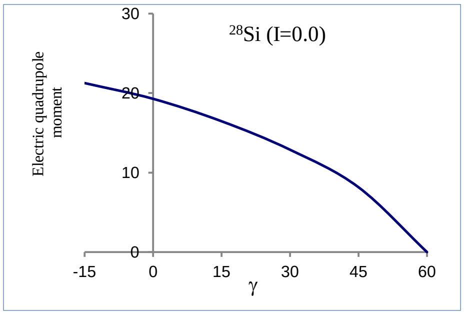

Fig. 20 The electric quadrupole moment as function of γ

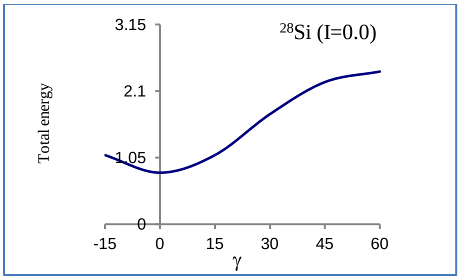

Fig. 21 The total energy as function of γ

Fourthly, for the nucleus ^{32}S

At fixed quadrupole deformation parameters $\varepsilon = 0.3$ and $\varepsilon_4 = -0.136$ we have

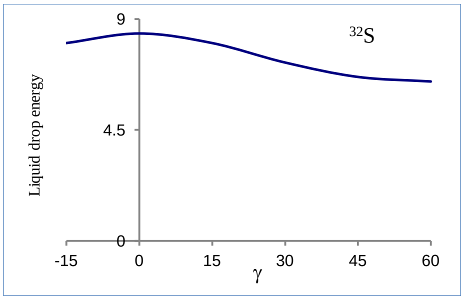

Fig. 22 The L.D. energy as function of γ

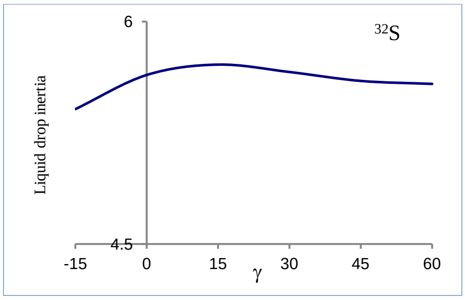

Fig. 23 The L.D. inertia as function of γ

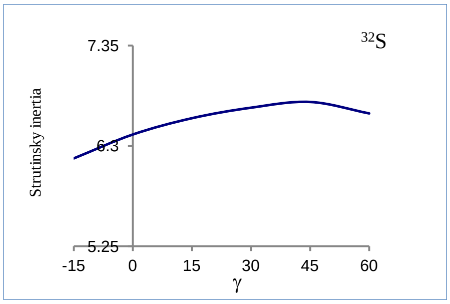

Fig. 24 The Strutinsky inertia function of γ

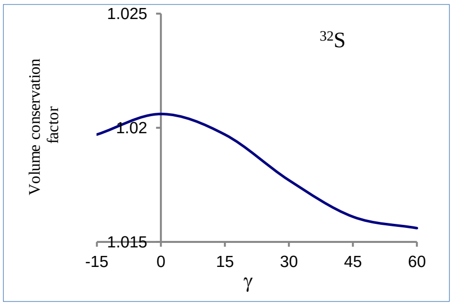

Fig. 25 The volume conservation factor as as function of γ

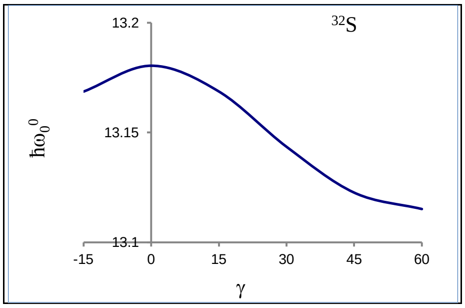

Fig. 26 The oscillator parameter $\hbar\omega_0^0$ as function of γ

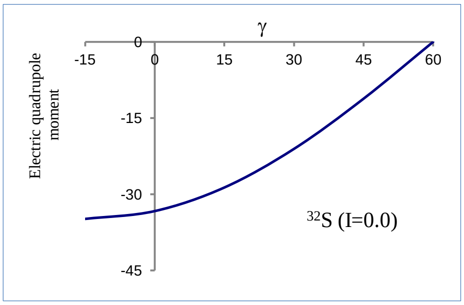

Fig. 27 The electric quadrupole moment as function of γ

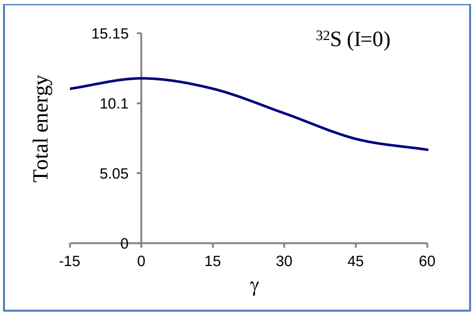

Fig. 28 The total energy as function of γ

Fifthly, for the nucleus ^{36}Ar

At fixed quadrupole deformation parameters $\varepsilon = -0.3$ and $\varepsilon_4 = 0$ we have

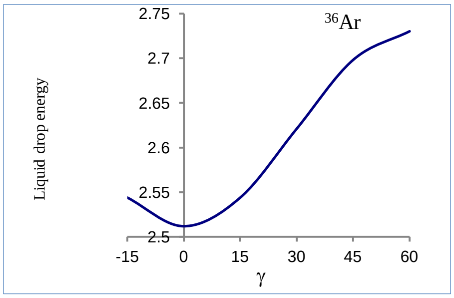

Fig. 29 The L.D.energy as function of γ

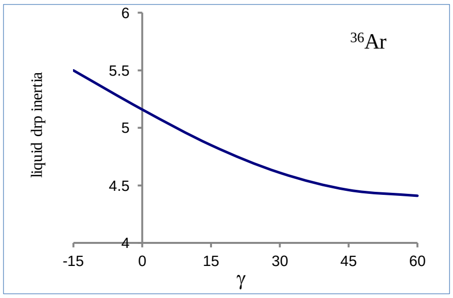

Fig. 30 The L.D.inertia as function of γ

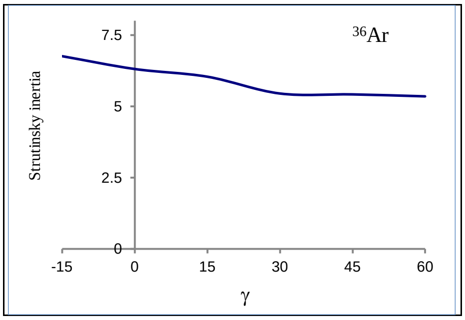

Fig. 31 The Strutinsky inertia as function of γ

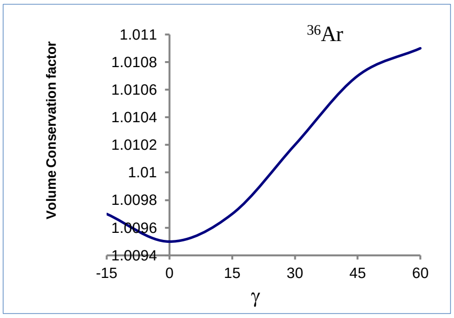

Fig. 32 The volume conservation factor as function of γ

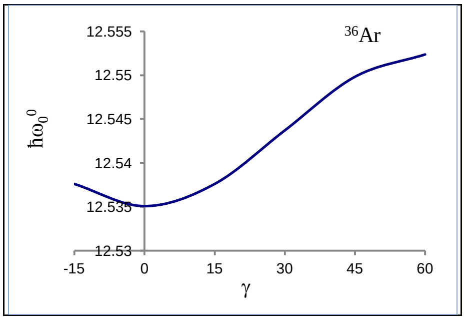

Fig. 33 The oscillator parameter $\hbar\omega_0^0$ as function of γ

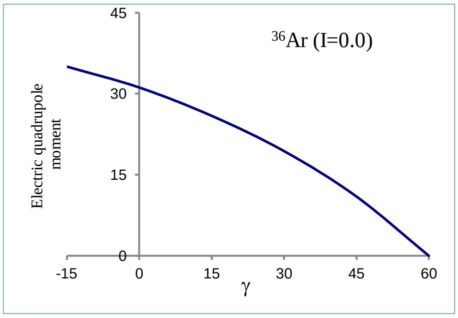

Fig. 34 The electric quadrupole moment as function of γ

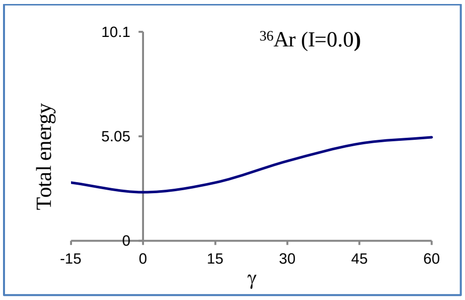

Fig. 35 The total energy as function of γ

In Table-1 we present the calculated values of the electric quadrupole moment (in barns) of the doubly even five nuclei ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S and ^{36}Ar according to formulas (3.5) and (3.6) together with the corresponding experimental values [26] and the value of the non axiality parameter γ, for which the calculated value is in better agreement with the corresponding experimental value.

Table-1 Electric quadrupole moment of the doubly even deformed nuclei in the s-d shell: ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S and ^{36}Ar by using the cranked Nilsson model.

Nuclei	Q_{exp}(barns)	Q_{cal}(barns)	γ
^{20}Ne	-0.23	-0.229	-15°
^{24}Mg	-0.166	-0.1648	30°
^{28}Si	0.165	0.1643	30°
^{32}S	-0.149	-0.1396	30°
^{36}Ar	0.11	0.1064	15°

In Table-2 we present the calculated values of the reciprocal moments of inertia of the nuclei ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S and ^{36}Ar by using the concept of the single-particle Schrödinger fluid for both of the cranking-and the rigid-body models. The values of the deformation parameter β, which produce the best values of the reciprocal moments of inertia, and the oscillator parameter $\hbar\omega_0^0$ are also given in Table-2.

It is clear from Table -1 that the results of the obtained electric quadrupole moment for the doubly even nuclei ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S and ^{36}Ar are in good agreement with the corresponding experimental values [26].

Table-2 Reciprocal moments of inertia of the nuclei ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S and ^{36}Ar by using the single-particle Schrödinger fluid.

Nucleus	^{20}Ne	^{24}Mg	^{28}Si	^{32}S	^{36}Ar
$\hbar\omega_0^0$ (MeV)	11.88	11.55	11.22	10.91	10.62
β	0.436	0.331	-0.233	0.487	0.324
$\frac{\hbar^2}{2\Im_{cr}}$ (KeV)	279.85	233.8	324.67	371.3	374.1
$\frac{\hbar^2}{2\Im_{rig}}$ (KeV)	172.32	69.86	57.97	41.96	152.3
$\frac{\hbar^2}{2\Im_{exp}}$ (KeV)	279.90	237.90	324.60	371	374

According to previous works [14, 15] the parameter β takes values in the interval $-0.50 \leq \beta \leq 0.50$ with a step 0.01. It is seen from Table-2 that the value of the deformation parameter β for the nucleus ^{28}Si is negative while the corresponding values for the four nuclei ^{20}Ne, ^{24}Mg, ^{32}S and ^{36}Ar are positive.

Furthermore, it is seen from Table-2 that the calculated values of the cranking-model reciprocal moments of inertia are in better agreement with the corresponding experimental values [27] rather than the rigid-body values. The disagreement between the values of the rigid-body reciprocal moment of inertia and the corresponding experimental values is due to the fact that the pairing correlation is not taken in concern in this model [13].

Moreover, it is seen from Figs. 4, 11, 18, 25 and 32 that the volume conservation factor does not depend mainly on the non axiality parameter γ and the variations are slowly for the five nuclei. The same conclusion holds also for the non deformed oscillator parameter $\hbar\omega_0^0$, as seen from Figs. 5, 12, 19, 26 and 33.

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