The Electric Quadrupole Moments of Some p-Shell Deformed Nuclei

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Abstract

The cranking Nilsson model is discussed, and its formulas are derived. Accordingly, we calculated the electric quadrupole moments of some deformed nuclei in the p-shell, namely: ^{6}Li, ^{7}Li, ^{9}Be, ^{10}B, ^{11}B, ^{12}B, ^{13}B, ^{11}C, and ^{14}N. Good results are obtained in comparison with the well-known experimental findings.

Keywords

Deformed nuclei; electric quadrupole moment; Cranked Nilsson model; p-shell nuclei.

1. Introduction

The nuclear collective rotation [1] is a topic of the nuclear structure theory, which has grown steadily, both in the sophistication of its theory and in the range of data to which it relates. The large values of the quadrupole moments observed in some nuclei, far away from closed shells, implied a collective deformation and thereby a rotational degree of freedom. The most central parameters of collective rotation are the moments of inertia [2-8] and the quadrupole moments [9-15] of deformed nuclei. Consequently, the study of the nuclear moments of inertia and the quadrupole moments of deformed nuclei are sensitive checks for the validity of the nuclear structure theories. Furthermore, it is well known that nearly all fully microscopic theories of nuclear rotation are based on or related in some way to the cranking model, which was introduced by Inglis [16] in a semi classical way, but it can be derived fully quantum mechanically, at least in the limit of large deformations, and not too strong K-admixtures ($K << I$). The cranking model has the following advantages [17]:

(I) In principle, it provides a fully microscopic description of the rotating nucleus. There is no introduction of redundant variables, therefore, we can calculate the parameters of the rotational inertia microscopically within this model and get a deeper insight into the dynamics of rotational motion.

(II) It describes the collective angular momentum as a sum of single-particle angular momenta. Therefore, collective rotation as well as single-particle rotation, and all transitions in between such as decoupling processes, are handled on the same footing.

(III) It is correct for very large angular momenta, where classical arguments apply.

A simple and widely used way to describe the change of the single-particle structure with rotation is given by the cranked Nilsson model (CNM) [17-25]. It is the method of calculating the shell correction energy that made it possible to do large-scale calculations where the nuclear potential-energy surface was explored in detail as a function of different deformation degrees of freedom. Important achievements in this field include the prediction of Super deformed high-spin states and terminating bands. The collective mode is more evident when one considers the excitation levels of the even-even nuclei. The excitations of even-even nuclei indicated that these nuclei have ground-state spin and parity 0^+ and the first excited state 2^+.

The quadrupole moment of an axially deformed nucleus is connected directly with its deformation parameter β and therefore with its moment of inertia. The quadrupole moments of these axially deformed nuclei are obtained as functions of their moments of inertia. Variations of the moments of inertia and the quadrupole moments of these nuclei in terms of the deformation parameters of these nuclei are also given. The obtained numerical results in the case of the cranking-model moments of inertia for all the considered nuclei and the rigid body-model moments of inertia, for some nuclei, are in good agreement with the corresponding experimental values, which showed that the assumption that these nuclei are deformed and have axes of symmetry is more reliable.

In the present paper we applied the CNM to the nuclear collective motion and to the calculations of the electric quadrupole moments of some deformed nuclei. Accordingly, the electric quadrupole moments of the deformed nuclei in the p-shell, namely the nuclei: ^{6}Li, ^{7}Li, ^{9}Be, ^{10}B, ^{11}B, ^{12}B, ^{13}B, ^{11}C, and ^{14}N are calculated.

2. Single Nucleon in a Deformed Nucleus and the Cranked Nilsson Model

The single particle Hamiltonian in the Cranked Nilsson model assumes the form [17-25]

$$H = H^{(0)} + H^{(1)} - \omega j_x, \tag{2.1}$$

where

$$H^{(0)} = \frac{p^2}{2m} + \frac{1}{2} m\{\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\}. \tag{2.2}$$

Here, the oscillator parameters ω_x, ω_y and ω_z assume the form [17]

$$\begin{aligned}
\omega_x &= \omega_0(\beta,\gamma)\left[1 - \left(\sqrt{\frac{5}{4\pi}}\beta\right) cos\left(\gamma - \frac{2\pi}{3}\right)\right],\\
\omega_y &= \omega_0(\beta,\gamma)\left[1 - \left(\sqrt{\frac{5}{4\pi}}\beta\right) cos\left(\gamma + \frac{2\pi}{3}\right)\right],\\
\omega_z &= \omega_0(\beta,\gamma)\left[1 - \left(\sqrt{\frac{5}{4\pi}}\beta\right) cos(\gamma)\right],
\end{aligned} \tag{2.3}$$

where β and γ are the quadrupole deformation degrees of freedom. The second term in the right-hand side of equation (2.1) is given by

$$H^{(1)} = 2\hbar\omega_0\sqrt{\frac{4\pi}{9}}\rho^2\varepsilon_4 V_4 + V', \tag{2.4}$$

where the stretched square radius ρ^2 is defined by

$$\rho^2 = \frac{m}{\hbar}\{\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\}, \tag{2.5}$$

The hexadecapole potential is defined to obtain a smooth variation [17-25] in the γ-plane so that the axial symmetry is not broken for $\gamma = -120^o, -60^o, 0^o$ and 60^o. It is of the form [17]

$$V_4 = a_{40}Y_{4,0} + a_{42}\left(Y_{4,2} + Y_{4,-2}\right) + a_{44}\left(Y_{4,4} + Y_{4,-4}\right), \tag{2.6}$$

where the a_{4i} parameters are chosen as

$$a_{40} = \frac{1}{6}(5\cos^2\gamma + 1), \qquad a_{42} = -\frac{1}{12}\sqrt{30}\sin 2\gamma, \quad a_{44} = \frac{1}{12}\sqrt{70}\sin^2\gamma.$$

and

$$V' = -\kappa(N)\hbar\omega_0^o\{2\ell_t.s + \mu(N)(\ell^2{}_t - \langle\ell^2{}_t\rangle_N)\}. \tag{2.7}$$

In equation (2.7) t refers to the stretched coordinates $\xi = x\sqrt{M\omega_x/\hbar}$ etc., and ε_4 in equation (2.4) refers to the hexadecapole deformations degree of freedom. j_x in equation (2.1) is the x-component of the total angular momentum J.

3. Derivations

3.1 The Hamiltonian $H^{(0)}$

The angular frequencies, equations (2.3), can be simplified to

$$\omega_x = \omega_0\left[1 + \frac{\varepsilon}{3}\left(\cos\gamma - \sqrt{3}\sin\gamma\right)\right], \tag{3.1}$$

$$\omega_y = \omega_0\left[1 + \frac{\varepsilon}{3}\left(\cos\gamma + \sqrt{3}\sin\gamma\right)\right], \tag{3.2}$$

$$\omega_z = \omega_0\left[1 - \frac{2}{3}\varepsilon\ \cos\gamma\right]. \tag{3.3}$$

Hence

$$\begin{aligned}\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2 &= \omega_0^2r^2 + \frac{2}{3}\varepsilon\cos\gamma\,\omega_0^2r^2(1 - 3\cos^2\theta)\\ &-\frac{2}{3}\varepsilon\sqrt{3}\sin\gamma\ \omega_0^2r^2\sin^2\theta\cos 2\varphi + \frac{\varepsilon^2}{9}\cos^2\gamma\ \omega_0^2r^2(1 + 3\cos^2\theta)\\ &-\frac{\varepsilon^2}{9}\sqrt{3}\sin^2 2\gamma\ \omega_0^2r^2\sin^2\theta\cos 2\varphi + \frac{\varepsilon^2}{9}\ 3\sin^2\gamma\ \omega_0^2r^2(1 - \cos^2\theta).\end{aligned} \tag{3.4}$$

Accordingly, the Hamiltonian $H^{(0)}$ takes the form

$$H^{(0)} = -\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\,\omega_0^2r^2 - \frac{m}{2}\left(\frac{2}{3}\varepsilon\cos\gamma\,\omega_0^2r^2\sqrt{\frac{16\pi}{5}}\ Y_{2,0}\right)$$

$$-\frac{m}{2}\left(\frac{2}{3}\varepsilon\sqrt{3}\,sin\,\gamma\;\omega_0^2 r^2\sqrt{\frac{8\pi}{15}}\;(Y_{2,2}+Y_{2,-2})\right)+\frac{m}{2}\left(\frac{\varepsilon^2}{9}cos^2\gamma\;\omega_0^2 r^2\left(\sqrt{\frac{16\pi}{5}}\,Y_{2,0}+2\right)\right)$$

$$-\frac{m}{2}\left(\frac{\varepsilon^2}{9}\sqrt{3}\,sin^2 2\gamma\;\omega_0^2 r^2\sqrt{\frac{8\pi}{15}}\;(Y_{2,2}+Y_{2,-2})\right)+\frac{m}{2}\left(\frac{\varepsilon^2}{9}\;3\;sin^2\gamma\;\omega_0^2 r^2\left(\sqrt{\frac{16\pi}{5}}\,Y_{2,0}-2\right)\right). \quad (3.5)$$

The two deformation parameters ε and δ are equal and they are related to the well-known deformation parameter β by the following relation [17]

$$\varepsilon=\delta=\frac{3}{2}\sqrt{\frac{5}{4\pi}}\,\beta. \quad (3.6)$$

The Hamiltonian $H^{(0)}$, then, takes the form

$$H^{(0)}=-\frac{\hbar^2}{2m}\nabla^2+\frac{1}{2}m\,\omega_0^2 r^2-\beta m\,\omega_0^2 r^2\,Y_{2,0}cos\,\gamma-\frac{\sqrt{2}}{2}\beta m\;\omega_0^2 r^2\;(Y_{2,2}+Y_{2,-2})\,sin\,\gamma$$

$$+\frac{5}{32\pi}\beta^2 m\,\omega_0^2 r^2\left(\sqrt{\frac{16\pi}{5}}\,Y_{2,0}+2\,cos\,\gamma-\sqrt{\frac{8\pi}{5}}\;(Y_{2,2}+Y_{2,-2})\,sin\,2\gamma\right). \quad (3.7)$$

Hence, to the first order in β the Hamiltonian $H^{(0)}$ takes the form

$$H^{(0)}=-\frac{\hbar^2}{2m}\nabla^2+\frac{1}{2}m\,\omega_0^2 r^2-\beta m\,\omega_0^2 r^2\,Y_{2,0}cos\,\gamma-\frac{\sqrt{2}}{2}\beta m\;\omega_0^2 r^2\;(Y_{2,2}+Y_{2,-2})\,sin\,\gamma. \quad (3.8)$$

3.2 The Hamiltonian $H^{(1)}$

Direct substitution for the different quantities in the operator ρ^2 gives

$$\rho^2=\frac{m\omega_0}{\hbar}r^2\left[1-\frac{\varepsilon}{3}cos\,\gamma\sqrt{\frac{16\pi}{5}}\,Y_{2,0}-\frac{\varepsilon}{3}\sqrt{3}\,sin\,\gamma\sqrt{\frac{8\pi}{15}}(Y_{2,2}+Y_{2,-2})\right]. \quad (3.9)$$

3.3 The term $2\hbar\omega_0\sqrt{\frac{4\pi}{9}}\rho^2\varepsilon_4 V_4$

It is not difficult to show that

$$2\hbar\omega_0\sqrt{\frac{4\pi}{9}}\rho^2\varepsilon_4 V_4=$$

$$\sqrt{\frac{16\pi}{9}}m\,\omega_0^2\varepsilon_4 r^2\begin{bmatrix}\frac{1}{6}(5\,cos^2\gamma+1)Y_{4,0}\pm\frac{1}{12}\sqrt{30}\,sin\,2\gamma\,(Y_{4,2}+Y_{4,-2})\\ +\frac{1}{12}\sqrt{70}\,sin^2\gamma(Y_{4,4}+Y_{4,-4})\end{bmatrix}$$

$$\times\left[-\frac{\varepsilon}{3}cos\,\gamma\sqrt{\frac{16\pi}{5}}\,Y_{2,0}\times\begin{bmatrix}\frac{1}{6}(5\,cos^2\gamma+1)Y_{4,0}\pm\frac{1}{12}\sqrt{30}\,sin\,2\gamma\,(Y_{4,2}+Y_{4,-2})\\ +\frac{1}{12}\sqrt{70}\,sin^2\gamma(Y_{4,4}+Y_{4,-4})\end{bmatrix}\right]$$

$$-\frac{\varepsilon}{3}\sqrt{3}\sin\gamma\sqrt{\frac{8\pi}{15}}\left(Y_{2,2}+Y_{2,-2}\right)\left[\begin{array}{c}\frac{1}{6}(5\cos^2\gamma+1)Y_{4,0}\pm\frac{1}{12}\sqrt{30}\sin 2\gamma\left(Y_{4,2}+Y_{4,-2}\right)\\ +\frac{1}{12}\sqrt{70}\sin^2\gamma\left(Y_{4,4}+Y_{4,-4}\right)\end{array}\right] \tag{3.10}$$

In the above equations, the oscillator frequency $\omega_0 = \omega_0(\delta)$ is given in terms of the nondeformed parameter ω_0^0 by the following relation [26]

$$\omega_0(\delta) = \omega_0^0\left(1-\frac{4}{3}\delta^2-\frac{16}{27}\delta^3\right)^{-\frac{1}{6}}. \tag{3.11}$$

For the non-deformed oscillator parameter $\hbar\omega_0^0$ we used the one which is given in terms of the mass number A, the number of neutrons N and the number of protons Z by [27]

$$\hbar\omega_0^0 = 38.6\, A^{-\frac{1}{3}} - 127.0\, A^{-\frac{4}{3}} + 14.75\, A^{-\frac{4}{3}}(N-Z). \tag{3.12}$$

4. The Single Particle Energy Eigenvalues and Eigenfunctions

The method of finding the energy eigenvalues and eigenfunctions of the Hamiltonian H can be illustrated as follows. Solve the Schrödinger's equation

$$H_0^{(0)}\psi_i^{(0)} = E_i^{(0)}\psi_i^{(0)}, \tag{4.1}$$

exactly to find the unperturbed energy eigenvalues $E_i^{(0)}$ and eigenfunctions $\psi_i^{(0)}$. Then modify the functions $\psi_i^{(0)}$ to become eigenfunctions for the solutions of the corresponding equation for $H_0^{(0)} + V'$. Hence, use the functions obtained in the last step to construct the complete function ψ, the eigenfunction of the Hamiltonian H, in the form of linear combinations of the above functions, as basis functions, with given total angular momentum j and parity π. Finally, construct the Hamiltonian matrix H by calculating its matrix elements with respect to the basis functions defined in the last step. Diagonalizing the Hamiltonian matrix H one finds the energy eigenvalues E_n and eigenfunctions ψ_n as functions of the non-deformed oscillator parameter $\hbar\omega_0^0$ and the parameters of the potentials. These methods can be explained in the following steps.

The solutions of the equation $H_0^{(0)}\psi_i^{(0)} = E_i^{(0)}\psi_i^{(0)}$, are given, with the usual notations, by [2,4]

$$\psi_i^{(0)} \equiv |N\ell\Lambda\rangle = R_{N\ell}(r)Y_{\ell\Lambda}(\theta,\varphi), \tag{4.2}$$

$$E_i^{(0)} = \varepsilon_N^0 = \left(N+\frac{3}{2}\right)\hbar\omega_0(\delta), \tag{4.3}$$

where $Y_{\ell\Lambda}(\theta,\varphi)$ are the normalized spherical harmonics with $\Lambda = -\ell, -\ell+1, \dots, 0, \dots, \ell-1, \ell$ and ℓ is the nucleon orbital angular momentum quantum number. The normalized radial wave functions $R_{N\ell}(r)$ are given by

$$R_{N\ell}(r) = a_0^{-\frac{3}{2}} \sqrt{\frac{2\Gamma\left(\frac{N-\ell+2}{2}\right)}{\Gamma\left(\frac{N+\ell+3}{2}\right)}}\, e^{-\frac{\rho^2}{2}} \rho^\ell L_{\frac{N-\ell}{2}}^{\ell+\frac{1}{2}}(\rho^2), \tag{4.4}$$

where $\rho = \frac{r}{a_0}$, $a_0 = \sqrt{\frac{\hbar}{m\omega_0(\delta)}}$ and the number of quanta of excitation N is related to the orbital angular momentum quantum number ℓ by $\ell = N, N-2, \dots, 0$ or 1.

The last function in the right-hand side of equation (4.4) is the associated Laguerre polynomial. Since the nucleon has spin $\frac{1}{2}$ and intrinsic spin wave functions $\chi_{s\Sigma}$, where $\Sigma = \pm\frac{1}{2}$, the single particle wave functions of the Hamiltonian $H^{(0)}$ are, then, given by

$$\psi_i^{(0)} \equiv |N\ell\Lambda\Sigma\rangle = R_{N\ell}(r) Y_{\ell\Lambda}(\theta,\varphi)\chi_{s\Sigma}. \tag{4.5}$$

Wave functions with given values of the number of quanta of excitations N, the orbital angular momentum quantum number ℓ, the total angular momentum J and the parity π can be constructed from the functions (4.5), in the usual manner [2,4,13-15], as follows

$$|N\ell\Lambda\Sigma\rangle = \sum_{\Lambda+\Sigma=\Omega} \left(\ell\Lambda, \tfrac{1}{2}\Sigma|J\Omega\right) |N\ell\Lambda\Sigma\rangle. \tag{4.6}$$

The functions $|N\ell J\pi\rangle$ are used as basis functions for the construction of the single particle nuclear wave functions with given total angular momentum J and parity π, in the usual manner, as follows

$$|N\ell J\pi\rangle = \sum_{N\ell} \sum_{\Lambda+\Sigma=\Omega} C_{N\ell} \left(\ell\Lambda, \tfrac{1}{2}\Sigma|J\Omega\right) |N\ell\Lambda\Sigma\rangle. \tag{4.7}$$

Accordingly, we obtain 15 wave functions, states, namely

$\left|\frac{1}{2}^+\right\rangle, \left|\frac{3}{2}^+\right\rangle, \left|\frac{5}{2}^+\right\rangle, \left|\frac{7}{2}^+\right\rangle, \left|\frac{9}{2}^+\right\rangle, \left|\frac{11}{2}^+\right\rangle, \left|\frac{13}{2}^+\right\rangle, \left|\frac{1}{2}^-\right\rangle, \left|\frac{3}{2}^-\right\rangle, \left|\frac{5}{2}^-\right\rangle, \left|\frac{7}{2}^-\right\rangle, \left|\frac{9}{2}^-\right\rangle, \left|\frac{11}{2}^-\right\rangle, \left|\frac{13}{2}^-\right\rangle$ and $\left|\frac{15}{2}^-\right\rangle$.

The matrix elements of the Hamiltonian $H_0^{(0)} + V'$ with respect to the functions (4.7) are given by

$$\left\langle J^\pi \middle| H_0^{(0)} + V' \middle| J^\pi \right\rangle = \sum_{N\ell N'\Lambda,\Sigma,\Lambda',\Sigma'} \left(\ell\Lambda, \tfrac{1}{2}\Sigma|J\Omega\right)\left(\ell\Lambda', \tfrac{1}{2}\Sigma'|J\Omega'\right)$$
$$\times\, C_{N\ell} C_{N'\ell} \Big[\left(N + \tfrac{3}{2}\right)\hbar\omega_0(\delta)\delta_{N,N'}\delta_{\Lambda,\Lambda'}\delta_{\Sigma,\Sigma'}\delta_{\Omega,\Omega'} - \chi\hbar\omega_0^0 \Big[(2\Lambda\Sigma + \mu\ell(\ell+1))\delta_{\Lambda,\Lambda'}\delta_{\Sigma,\Sigma'} + \sqrt{(\ell-\Lambda)(\ell+\Lambda+1)}\delta_{\Lambda+1,\Lambda'}\delta_{\Sigma-1,\Sigma'} + \sqrt{(\ell+\Lambda)(\ell-\Lambda+1)}\delta_{\Lambda-1,\Lambda'}\delta_{\Sigma+1,\Sigma'}\Big]\delta_{N,N'}\delta_{\Omega,\Omega'}\Big]. \tag{4.8}$$

The matrix elements of the operator r^2 with respect to the basis functions $|N\ell\Lambda\Sigma\rangle$ are given, with the usual notations [2,4], by

$$\langle N\ell\Lambda\Sigma|r^2|N'\ell\Lambda'\Sigma'\rangle = a_0^2\left[\left(N+\tfrac{3}{2}\right)\delta_{N,N'} + \sqrt{n\left(n+\ell+\tfrac{1}{2}\right)}\ \delta_{N-Z,N'} + \sqrt{(n+1)\left(n+\ell+\tfrac{3}{2}\right)}\ \delta_{N+Z,N'}\right]\delta_{\Lambda,\Lambda'}\delta_{\Sigma,\Sigma'}\ , \tag{4.9}$$

where $a_0^2 = \frac{\hbar}{m\omega_0(\delta)}$ and $N = 2n+\ell$.

The matrix elements of the spherical harmonics $Y_{LM}(\theta,\varphi)$ with respect to the basis functions $|N\ell\Lambda\Sigma\rangle$ are given by [2,4]

$$\langle N\ell\Lambda\Sigma|Y_{LM}|N'\ell\Lambda'\Sigma'\rangle = (-1)^{2\ell+1}\sqrt{\tfrac{2L+1}{4\pi}}\begin{pmatrix}\ell & L & \ell\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\ell & L & \ell\\ -\Lambda & M & \Lambda'\end{pmatrix}\delta_{\Sigma,\Sigma'}. \tag{4.10}$$

5. Total Nuclear Quantities

We define the following total nuclear quantities [17]

$$E_{sp} = \sum_{occ} e_i = \sum_{occ} e_i^{\omega} + \hbar\omega\sum_{occ} m_i, \tag{5.1}$$

$$I = \sum_{occ} m_i, \tag{5.2}$$

where the summation is carried out over the occupied orbitals in a specific configuration of the nucleus. The shell energy is now calculated from [17]

$$E_{shell}(I) = E_{sp}(I) - \langle E_{sp}(I)\rangle, \tag{5.3}$$

where $\langle E_{sp}(I)\rangle$ is the smoothed single-particle sum evaluated according to the Strutinsky prescription [28-29]. The detailed formulas for $\langle E_{sp}(I)\rangle$ are discussed in [17,28] for $I = 0$, and in [28-29] for $I \neq 0$.

The pairing energy is an important correction that should decrease with increasing spin and become essentially unimportant at very high spins. To obtain an $(I = 0)$ average pairing gap Δ, which varies as $A^{-\frac{1}{2}}$, the pairing strength G is chosen as [17-25]

$$G_{p,n} = \frac{1}{A}\left(g_0 \pm g_1\frac{N-Z}{A}\right)\ \text{(MeV)}, \tag{5.4}$$

with $\frac{g_1}{g_0} \sim \frac{1}{3}$. Furthermore, the number of orbitals included in the pairing calculation should vary as $\sqrt{Z}$ and $\sqrt{N}$ for protons (p) and neutrons (n), respectively.

The total nuclear energy is now calculated by replacing the smoothed single-particle sum by the rotating-liquid-drop energy and adding the pairing correction

$$E_{tot}(\bar{\varepsilon},I) = E_{shell}(\bar{\varepsilon},I) + E_{RLD}(\bar{\varepsilon},I) + E_{pair}(\bar{\varepsilon},I), \tag{5.5}$$

or

$$E_{tot}(\bar{\varepsilon},I) = E_{sp} - A + E_{LD} - BI^2 + \frac{\hbar^2}{2\Im_{rig}} \tag{5.6}$$

where $\bar{\varepsilon} = (\varepsilon, \gamma, \varepsilon_4)$, E_{LD} is liquid drop energy, $A = \langle E_{sp}(I) \rangle$ and B is the smooth moment of inertia factor, $B = \frac{\hbar^2}{2\Im_{Strut}}$. The shell and pairing energies are evaluated separately for protons and neutrons at $I = 0$, while the renormalization of the moment of inertia introduces a coupling when evaluating E_{shell} for $I > 0$. In the computer program, E_{pair} is included only for $I = 0$. The protons and neutrons are also coupled through the requirement that the shape of the respective potentials and the rotational frequencies are identical.

In the liquid drop model [17-25], the nuclear mass is given by

$$E_{L.D.} = -a_V \left(1 - \kappa_V \left(\frac{N-Z}{A}\right)^2\right) A + \frac{3}{5}\frac{e^2 Z^2}{R_c}\left[B_c(\bar{\varepsilon}) - \frac{5\pi^2}{6}\left(\frac{d}{R_c}\right)^2\right]$$

$$+a_s \left(1 - \kappa_s \left(\frac{N-Z}{A}\right)^2\right) A^{2/3} B_s(\bar{\varepsilon}) + \begin{cases} +12/\sqrt{A} & odd - odd\ nuclei \\ 0 & odd - even\ nuclei \\ -12/\sqrt{A} & even - even\ nuclei\,. \end{cases} \qquad (5.7)$$

In this formula, $B_c(\bar{\varepsilon}) = B_{coul}(\bar{\varepsilon})/B_{coul}(\bar{\varepsilon} = 0)$ and $B_s(\bar{\varepsilon}) = B_{surf}(\bar{\varepsilon})/B_{surf}(\bar{\varepsilon} = 0)$, are the Coulomb and surface energies of a nucleus with a sharp surface in units of their corresponding values for spherical shape. The second term in the Coulomb energy is a (shape-independent) diffuseness correction with d being the diffuseness. The Coulomb energy constant is often defined as $a_c = (3/5)(e^2/R_c)$. When calculating the nuclear mass, one should note that the average pairing energy should be subtracted from E_{pair}.

The calculation of the Coulomb correction is somewhat involved: the original six-dimensional integral can be simplified only to four dimensions [17]. Furthermore, the use of stretched coordinates leads to complicated expressions.

Because of the incompressibility of nuclear matter, the nuclear volume is kept constant when the nucleus is deformed. This is achieved by varying the frequency $\omega_0(\varepsilon, \gamma, \varepsilon_4)$ from its value for a spherical shape, ω_0^0. The integration of the nuclear volume is most easily performed in the stretched-coordinate system and then multiplied with the corresponding Jacobian, a constant proportional to $\sqrt{\omega_x \omega_y \omega_z / \omega_0^3}$.

From the single-particle wave functions the electric (or mass) quadrupole moment may be calculated as

$$Q_2 = \sum_{occ} p_i \langle \chi_i^{\omega} | q_2 | \chi_i^{\omega} \rangle, \qquad (5.8)$$

where $p_i = 1$ for protons and 0 for neutrons and the functions χ_i^{ω} are those which are given by equation (4.7).

The relation between the measured quadrupole moment, denoted by Q_S and Q_2 is given by [26]

$$Q_S = \frac{3K^2 - I(I+1)}{(I+1)(2I+2)} Q_2. \qquad (5.9)$$

The number I in equation (5.9) is the spin-quantum number of the specified nuclear state and K is its component along the body-fixed z-axis. It turns out that always the ground state spin of the nucleus $I_0 = \Omega = K$, where Ω is the z-component of the total angular momentum J, except when $\Omega = \frac{1}{2}$, in which case the ground state spin I_0 is given as function of the decoupling factor a, as given by Table-III of reference [26].

6. Results and Discussions

We have applied the CNM to the nine deformed nuclei in the p-shell, namely the nuclei ^{6}Li, ^{7}Li, ^{9}Be, ^{10}B, ^{11}B, ^{12}B, ^{13}B, ^{11}C, and ^{14}N in order to calculate the intrinsic electric quadrupole moments of these nuclei and as a consequence the measured quadrupole moment of the nine nuclei are then obtained.

In Table-1 we present the calculated values of the quadrupole moments of the nine deformed nuclei, together with the corresponding experimental values [30,31] and the values of the deformation parameters for which the obtained results are in good agreement with the corresponding experimental values. The values of the nondeformed oscillator parameter $\hbar\omega_0^0$, the total spin and parity I^π are also given in this table.

Table-3 Quadrupole moments of the nine deformed nuclei

Nucleus	β	γ°	I^π	$\hbar\omega_0^0$ (MeV)	Q_s CNM (e m barns)	Q exp. [30-31] (e m barns)
^{6}Li	-0.12	10°	1^+	9.5939	-0.079	-0.08
^{7}Li	-0.18	γ°	$\frac{3}{2}^-$	11.7960	-3.935	-4.0
^{9}Be	0.27	γ°	$\frac{3}{2}^-$	12.5610	4.921	5.0
^{10}B	0.44	30°	3^+	12.0220	8.441	8.5
^{11}B	0.37	30°	$\frac{3}{2}^-$	12.7680	4.009	4.1
^{12}B	0.18	γ°	1^+	13.3110	1.823	1.8
^{13}B	0.39	30°	$\frac{3}{2}^-$	13.709	4.895	±5.0
^{11}C	0.21	30°	$\frac{3}{2}^-$	11.5620	3.002	3.1
^{14}N	0.12	10°	1^+	12.2520	0.936	1.0

The analysis of the quadrupole moments of the considered nuclei shows that, among all the considered nuclei, the nuclei ^{6}Li and ^{7}Li have, only, oblate shapes while the other nuclei have prolate deformations. Moreover, it is seen from Table-1 that the obtained values of the electric quadrupole moments of the nine deformed nuclei are in good agreement with the corresponding experimental values.

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